Exploitation of Inertial Coupling in Passive Gravity-Gradient-Stabilized Satellites

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A passive, three-axes stabilization system is described which relies solely on the gravity gradient to provide both the damping and the restoring torques for a vertically oriented satellite. Damping is effected by the energy dissipation of any relative motion that exists between the main satellite body and a suitably connected auxiliary body. It is shown that a single auxiliary body having a single degree of freedom can be positioned to introduce inertial coupling that permits the attitude motions about all three axes to be damped. An analysis is presented which leads to the solution of optimum system parameters for an example configuration of this type.

Nomenclature

$\bar{o}_j (j = 1, 2, 3)$	= unit vectors defining orthogonal reference frame fixed in satellite orbit such that $\bar{o}_3$ is directed toward center of earth, $\bar{o}_1$ is in direction of motion, and $\bar{o}_2$ completes the right-hand set
$\bar{v}_j (j = 1, 2, 3)$	= unit vectors defining an orthogonal reference frame fixed in main satellite body; in steady state $\bar{v}_j = \bar{o}_j (j = 1, 2, 3)$
$\bar{c}_j (j = 1, 2, 3)$	= unit vectors defining orthogonal reference frame fixed in main satellite body; $\bar{c}_3 = \bar{v}_3$, oriented relative to $\bar{v}_j$ frame by rotation through angle δ about $\bar{v}_3$ axis
$\bar{d}_j (j = 1, 2, 3)$	= unit vectors defining orthogonal reference frame fixed in damper rod; in steady $\bar{d}_j = \bar{c}_j (j = 1, 2, 3)$
$\bar{b}_j (j = 1, 2, 3)$	= unit vectors along principal inertia axes of main satellite body
T	= torque
$I_j (j = 1, 2, 3)$	= principal inertias of main satellite body
$I_{ij} (i, j = 1, 2, 3)$	= elements of inertia matrix of main satellite body expressed relative to $\bar{v}_j$ frame
I_d	= maximum inertia of damper rod
J_3	= mass distribution parameter, $(I_2 - I_1)/I_3$
α, β, ζ	= Euler angle sequence describing position of principal inertia axes of main satellite body ($\bar{b}_j$ system of axes) relative to reference frame fixed in satellite orbit ($\bar{o}_j$ system of axes). For small angles, α is analogous to θ, β to φ, and $\zeta - \gamma$ to ψ
$\Delta\zeta$	= yaw error $(\zeta - \gamma)$
φ, θ, ψ	= small rotation angles about 1, 2, and 3 axes, respectively
δ	= angle defining orientation of $\bar{c}_j$ frame relative to $\bar{v}_j$ frame, alternatively equilibrium angle between damper rod and orbital frame (positive value of δ is positive rotation about $\bar{v}_3$)
γ	= angle defining orientation of $\bar{b}_j$ frame relative to $\bar{v}_j$ frame, alternatively equilibrium angle between principal pitch inertia axis of main satellite body ($\bar{b}_1$ axis) and orbital plane (positive value of γ is positive rotation about $\bar{v}_3$)
σ_{max}	= maximum real part of roots of characteristic equation
D_2, D_3	= damping constants associated with angular rate dampers located on $\bar{c}_2$ and $\bar{c}_3$ axes, respectively
K_2, K_3	= spring constants associated with springs that restrain rotations of damper rod about $\bar{c}_2$ and $\bar{c}_3$ axes, respectively
B_2, B_3	= dimensionless damping constants $D_2/\omega_o I_d$ and $D_3/\omega_o I_d$, respectively
C_2, C_3	= dimensionless spring constants $K_2/\omega_o^2 I_d$ and $K_3/\omega_o^2 I_d$, respectively
ω_o	= orbital angular velocity
$\omega_i (i = 1, 2, 3, 4)$	= oscillatory angular velocities
t	= real time
τ	= dimensionless time $(\tau = \omega_o t)$

Suffixes

d	= either damper rod or value of variable expressed in $\bar{d}_j$ frame
v	= either main satellite body or value of variable expressed in $\bar{v}_j$ frame
o	= either orbital reference frame or value of variable expressed in $\bar{o}_j$ frame
c	= value of variable expressed in $\bar{c}_j$ frame
dc, etc.	= variable describing some relationships between $\bar{d}_j$ and $\bar{c}_j$ frame, etc.

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Introduction

It is well known that a satellite having unequal principal moments of inertia has torques acting on it because of the gradient of the earth's gravitational field. These torques tend to align the axis of least moment of inertia with the local vertical. Therefore, in principle, the gravity-gradient torques can be used as the basis of a stabilization system for an earth-pointing satellite.[1,2]

A successful gravity-gradient-stabilized satellite, in addition to having a preferred orientation in the gravitational field, must employ some method of damping oscillatory motions. This can be achieved passively if a secondary interaction can be found between the satellite and some field of force that produces torques having the correct phase relationship with the primary gravity-gradient torques. In principle, the secondary interaction can be produced by any field of force, provided that the rate of change of the field vectors is sufficiently small. Stabilization systems have been proposed which derive their damping from one or more of the three major force fields, that is, gravitational, magnetic, or solar electromagnetic radiation. In fact, the first passive gravity-gradient-stabilized satellite flown derived its damping from both the gravitational and magnetic fields and proved the feasibility of the technique at an orbital altitude of 400 naut miles.[3]

There are advantages in using the gravitational field for the secondary interaction in preference to either the magnetic or solar radiation fields. The latter fields are not geocentric, so that any torques derived from them are dependent not only on the satellite attitude but also on the position in the orbit. Therefore, these torques, in addition to providing the damping, always excite the satellite and introduce steady-state attitude errors. A further disadvantage of the magnetic field is that it is influenced by solar activity at high orbital altitudes. This may result in variations of both field strength and direction, thereby causing uncertainty in the damping and additional attitude errors.

The only possible way to use the gravitational field to damp the satellite motions is to adjoin one or more auxiliary bodies to the main satellite body so that motion can occur between them. The auxiliary bodies are oriented so that suitable phase differences exist between the "secondary" gravity-gradient torques acting upon them and the "primary" gravity-gradient torques on the main satellite body. These phase differences cause relative motion between the bodies in response to any disturbance, and the disturbance energy is dissipated by damping the relative motion. A practical approach offering the possibility of three-axis stabilization has been developed by Kamm,[4] who has made two significant contributions: The first is an ingenious and practical method of producing manifold increases in moment of inertia, and therefore in gravity torque, with a small expenditure of satellite mass, and the second is the effective use of two auxiliary bodies in the form of long rods that lie in the horizontal plane when in equilibrium. One of these rods is aligned with the velocity vector and damps oscillations within the orbital plane (pitch damping). The second damper rod is normal to the first and, because of gyroscopic coupling, permits damping of the other two degrees of rotational freedom (roll and yaw).

This paper presents preliminary results from a research program initiated to improve passive damping techniques. The objectives of this program are to reduce the potential weight and complexity of passive systems and, at the same time, to improve performance. For the type of system devised by Kamm,[4] it will be shown that both objectives can be accomplished if a damper rod is positioned to introduce inertial asymmetry relative to the orbital plane. The inertial asymmetry causes all of the satellite motions to be coupled and permits the oscillatory energy from all disturbances to be removed by a single rod and damping mechanism. An example configuration is considered in detail to illustrate the method of coupling the satellite motions inertially. The linearized equations of motion for circular orbits for the example configuration are given, as well as a procedure for finding optimum configurations of this type. The steady-state motion of optimum configurations caused by orbital eccentricity and example-time histories following an initial tumbling motion are also presented. These results were obtained by integrating the complete equations of motion. The digital computer program to accomplish this task was developed by the Rand Corporation.

Example Configuration

The philosophy used to provide three-axis passive stabilization for an earth-oriented satellite is shown by an example configuration that is analyzed in detail. The configuration chosen is one that shows promise but is not necessarily the optimum of the class of satellites that can be evolved to exploit inertial coupling.

An example of the physical arrangement of the type of configuration to be analyzed is shown in Fig. 1. The payload is assumed to be a cylinder with its axis lying in the horizontal plane and skewed out of the orbital plane. In orbit, slender rods are erected normal to the axis of the cylinder. In the equilibrium orientation, these rods lie along the local vertical. A damper rod, which is connected to the satellite through springs and dampers, lies in the horizontal plane and is skewed out of the orbital plane in a direction opposite to that of the axis of the cylinder. The physical arguments motivating the choice of this type of arrangement are given in the subsequent paragraphs.

The mass distribution required to produce restoring torques about all three axes can be reviewed if the entire

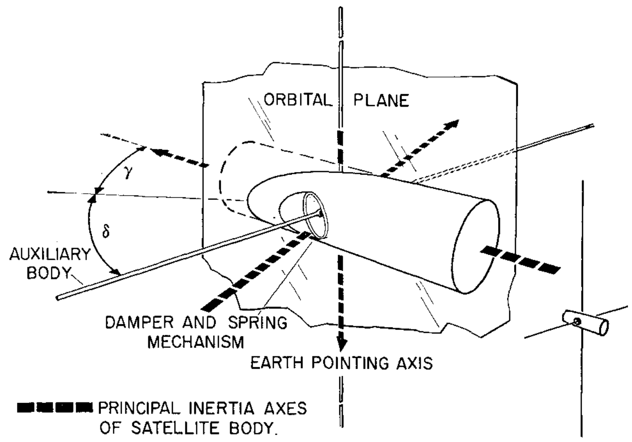


Fig. 1 Example configuration.

configuration (including the damper rod) shown in Fig. 1 is considered as a single rigid body. If a useful gravity-gradient restoring torque is to exist, the principal moment of inertia about the earth-pointing axis must be considerably less than that about any transverse axis. In this paper, the large differences required between moments of inertia are envisioned as being produced by the method devised by Kamm,[4] which consists in erecting a slender rod in the proper direction. In the example configuration, a rod is erected on either side of the payload along the axis intended to point earthward. The orientation about the earth-pointing axis is determined by gyroscopic torques caused by orbital motion. These torques act in a direction tending to align the principal axis of the largest moment of inertia with the normal to the orbital plane. In the example configuration, the transverse moment of inertia of the cylinder is chosen to be $1\frac{1}{2}$ times that of the damper rod. One of the principal moments of inertia of the entire configuration will, therefore, always be the largest regardless of the angle between cylinder axis and damper rod. If this angle is different from 90°, the axes of the cylinder and damper rod are both skewed to the orbital plane when the configuration is in equilibrium.

Two motions of the damper rod relative to the main satellite body are considered. The first is one in which two degrees of rotational freedom are permitted—one about the vertical (yaw) and the other, orthogonal to both the vertical and rod axes (roll/pitch). In the second, the yaw degree of freedom is removed, leaving the single roll/pitch degree of freedom.

When in equilibrium, the damper rod lies in the horizontal plane. In this position the rod is unstable in the gravitational field and seeks alignment with the local vertical. Consequently, a spring is required to stabilize the damper rod relative to the satellite. The reason for placing the damper rod in the horizontal plane can be seen intuitively from the motion of two rods, both aligned with the vertical and connected by a viscous damper. If one of the rods is disturbed, the relative motion between them will be damped until they are both oscillating with the same phase and amplitude. This mode of motion is undamped. The greatest possible damping, therefore, might be expected when the damper rod lies in the horizontal plane, although it must be stabilized relative to the satellite by means of a spring. This fact was observed by Kamm[4] and proved by Zajac[5] for simple pitch motion within the orbital plane.

Outwardly, it might appear that the damper rod is skewed to the orbital plane so that relative motion, and therefore damping, exists for pitching motion in the orbital plane or for rolling motion about the velocity vector. This reason alone, however, is not adequate to explain fully the physical situation, since it can then be argued that there is always a combination of pitch and roll motion which causes rotation about the rod axis. No relative motion results, in this instance, between the damper rod and satellite, and this motion therefore represents an undamped mode. The reason the undamped mode cannot occur is that skewing the damper rod to the orbital plane promotes inertial coupling. Coupling always exists, provided that those principal axes of the satellite body located in the horizontal plane and the damper rod axis are skewed to the orbital plane. Under these conditions, all motions are strongly coupled, so that motion about one axis induces motion about all others, thus permitting damping of all modes of motion by absorbing the energy between the satellite and a single damper rod.

Similar inertial properties can be obtained with arrangements quite different in appearance from that sketched in Fig. 1. For example, if the payload-mass distribution is spherical instead of cylindrical, the required over-all mass distribution can be realized if fixed rods are erected in the form of an X with the payload at the center. The auxiliary body, or damper rod, is then placed in the horizontal plane, as shown in the sketch. The earth-pointing axis now bisects the acute angles formed by the X, and the plane of the X is skewed to the orbital plane. The advantages of the latter are obvious if a sensor or antenna is to have an unobstructed view of the earth.

Analysis

A cursory analysis of the example configuration verified that sufficient coupling exists to permit damping of all modes of motion with a single auxiliary body. Following this initial step, optimum system parameters were defined on the basis of a linear analysis. The large-angle motion of optimum configurations was then studied by integration of the complete equations of motion.

Linear Analysis

The objective of the analysis is to select, from myriad possibilities, the one set of inertial parameters, spring constants, and damping constants which will yield optimum performance. Three performance criteria are considered in defining an optimum: 1) the motion must damp rapidly following an arbitrary disturbance, 2) the static restoring torque must be sufficient to limit errors from secular disturbance torques to acceptable levels, and 3) the response to periodic disturbances encountered in orbit must not be excessive.

The analysis is based on the solutions of linearized equations of motion for circular orbits. This approach is the most convenient for selecting an optimum configuration. Its use is justifiable since, after capture, the motion of any successful system will be within the small-angle range, where the linear approximation is valid. The transformed linearized equations of motion for the example configuration are given below in matrix form. A glossary of the notation, and a definition of the systems of axes are given in the Nomenclature. The time scale in these equations has been changed from t to $\tau = \omega_o t$ to express all results in terms of orbital frequency.

$$\begin{bmatrix} [-(s^2-4)\sin\delta] & [(s^2-3)\cos\delta] & [(2\sin\delta)s] & \left[\begin{matrix} s^2 - B_2 s - \\ (3+\sin^2\delta + C_2)\end{matrix}\right] & [(2\sin\delta)s] \\ [(2\sin^2\delta)s] & [-(\sin 2\delta)s] & [s^2+\cos 2\delta] & [-(2\sin\delta)s] & \left[\begin{matrix} s^2 - B_3 s + \\ (\cos 2\delta - C_3)\end{matrix}\right] \\ [s^2+4J_{11}] & \left[\frac{I_{12}}{I_{11}}(s^2-3)\right] & [(J_{11}-1)s] & \left[\begin{matrix} -\left[B_2\left(\frac{I_d}{I_{11}}\right)\sin\delta\right]s + \\ \left(\frac{I_{12}}{I_{11}}\right)\cos\delta - \\ C_2\left(\frac{I_d}{I_{11}}\right)\sin\delta \end{matrix}\right] & [0] \\ \left[\frac{I_{12}}{I_{22}}(s^2-4)\right] & [s^2+3J_{22}] & \left[-2\left(\frac{I_{12}}{I_{11}}\right)s\right] & \left[\begin{matrix} \left[B_2\left(\frac{I_d}{I_{22}}\right)\cos\delta\right]s + \\ \left(\frac{I_{12}}{I_{22}}\right)\sin\delta + \\ C_2\left(\frac{I_d}{I_{22}}\right)\cos\delta \end{matrix}\right] & [0] \\ [-(J_{33}-1)s] & \left[2\left(\frac{I_{12}}{I_{33}}\right)s\right] & [s^2+J_{33}] & [0] & \left[\left(\frac{I_d}{I_{33}}\right)(B_3 s + C_3)\right] \end{bmatrix} \begin{bmatrix} \varphi_{vo}(s) \\ \theta_{vo}(s) \\ \psi_{vo}(s) \\ \theta_{dc}(s) \\ \psi_{dc}(s) \end{bmatrix} = \begin{bmatrix} \frac{T_{d_2}(s)}{\omega_o^2 I_d} \\ \frac{T_{d_3}(s)}{\omega_o^2 I_d} \\ \frac{T_{v_1}(s)}{\omega_o^2 I_{11}} \\ \frac{T_{v_2}(s)}{\omega_o^2 I_{22}} \\ \frac{T_{v_3}(s)}{\omega_o^2 I_{33}} \end{bmatrix} \quad (1)$$

where

$$J_{11} = \frac{I_{22} - I_{33}}{I_{11}} \qquad J_{22} = \frac{I_{11} - I_{33}}{I_{22}} \qquad J_{33} = \frac{I_{22} - I_{11}}{I_{33}} \quad (2)$$

As in nearly all system studies, the optimum represents a compromise between several conflicting requirements. For instance, the passive stabilization system that is optimum solely from the standpoint of pointing error caused by a static disturbance torque would consist of a single slender rod with no damper rod and, therefore, could not be damped. The optimization procedure was thus devised to provide the variation of the more important responses with damper-rod moment of inertia. From these results, the damper-rod size that best suits all the requirements can be selected.

The first phase in the optimization procedure involves maximizing the damping of the least damped mode of the torque-free or transient motion. This was accomplished by use of an adaptation of the method of "steepest descent." In this method, a stationary point is sought such that any change of the system variables will always cause a reduction in the damping of the least damped mode. This stationary point is assumed to represent optimum damping. This first phase of the optimization procedure was completed for a range of damper rod inertias that vary from 0 to 24% of the moment of inertia of the satellite about the roll axis.

In the second phase of the optimization, the response to static and oscillatory disturbance torques was determined. The set of system parameters used for each selected ratio of damper rod inertia to satellite roll inertia was that determined from the first phase. No effort was made to reverse the process and seek smaller responses to the static or oscillatory torques at the expense of the damping of the torque-free transient motion.

It is emphasized that the entire analysis was limited to the example configuration for which $I_3 = 1.5\, I_d$. This choice is arbitrary and not necessarily an optimum. It is known, however, that the results are superior to those for $I_3 = I_d$.

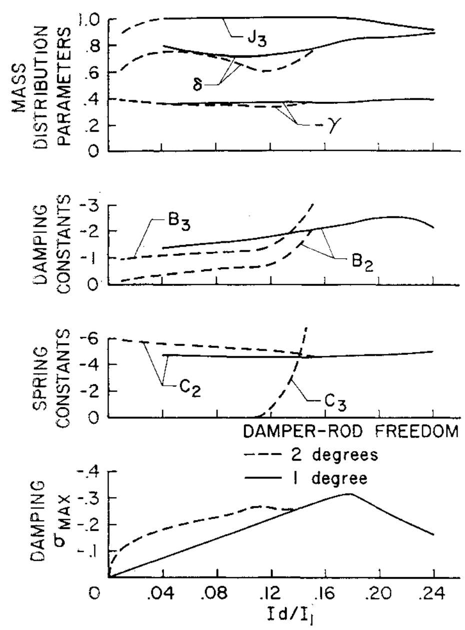


Fig. 2 Damping of least-damped mode and associated system parameters.

Large-Angle Motion

The results of a linear analysis leave unanswered such questions as: Is stabilization possible from an initial large error? Is it possible to achieve stabilization from an initial tumbling motion? What are the effects of orbital eccentricity? These questions can be answered by studying the time histories of satellite motion obtained from integration of the complete equations of motion. A digital computer program to accomplish this task has been developed by the Rand Corporation under a contract with NASA. This program incorporates logic that permits the damper-rod angular motion to be limited. For these studies the damper motion is assumed to be limited to $\pm 30°$.

Results and Discussion

Linear Analysis

An unexpected discovery of this study is that allowing the damper rod the additional degree of freedom about the yaw axis does not improve the maximum damping of the least damped mode of the transient motion. This is illustrated in Fig. 2, where the damping parameter σ_{max} and the associated optimum system parameters are plotted for various values of the damper-rod inertia ratio I_d/I_1. It is interesting that the study of the two-degree-of-freedom system, in itself, leads to the conclusion that the yaw degree of freedom can be eliminated without reducing the damping. Observe that as I_d/I_1 exceeds about 0.11, the optimum yaw spring becomes increasingly stiff (large negative value of C_3). Further, when I_d/I_1 is greater than about 0.15, the damping is insensitive to large increases in the stiffness of the yaw spring. This implies that the yaw degree of freedom can be eliminated without impairing the damping of the transient motions in this range of damper-rod inertia ratios.

The maximum damping of the least damped mode occurs when the damper-rod inertia ratio is about 0.18. The damping is sufficient to reduce the amplitude of any transient motion to about 15% of the initial amplitude in one orbit. However, a damper-rod inertia ratio considerably less than 0.18 might be selected in order to improve the response to specific disturbances. This is discussed later. When I_d/I_1 is less than 0.12, the damping of the system is clearly superior than when two degrees of damper-rod freedom are allowed. However, observe that the yaw spring (as represented by the constant C_3) must be very weak compared to the other spring C_2). A representative value for C_2, compatible with a typical satellite application, is about 100 dyne-cm/deg. Since the required C_3 spring is about 100 times weaker than the C_2 spring, its mechanical implementation may be a formidable problem. Further, the weaker spring is required to oppose a constant gyroscopic torque that tends to force the damper rod toward the orbital plane. If the spring is preloaded to furnish this torque, the initial offset from the zero-torque position must be of the order of several radians. In view of these mechanical considerations, it is concluded that a practical, reliable system is one that uses only a single degree of damper-rod freedom. The remainder of the discussion is, therefore, concerned exclusively with the performance of the single-degree-of-freedom damper.

The results shown in Fig. 2 indicate the size of the real part of the root of the characteristic equation which represents the least damped mode of motion. The magnitude of the real parts of all the roots of the characteristic equation is shown in Fig. 3. The envelope of the maxima of the real parts of all the roots in Fig. 3 corresponds to the curve of Fig. 2. The roots always occur as four complex pairs, so that only four distinct real parts exist. In Fig. 3, these real parts have been ordered according to the magnitude of the corresponding imaginary parts. At any value of I_d/I_1, there are always three distinct pairs of roots which have equal real parts. When the damping of the least damped mode is a maximum, near $I_d/I_1 = 0.18$, it appears that real parts of all four roots are equal. The computer technique employed to obtain the optima is not sufficiently accurate to verify that this does occur exactly, and the authors have been unable to verify this fact by analytical approaches. However, Zajac[5] has shown that an analogous situation exists in the case of a simple pitch damper. In this case the characteristic equation is a fourth-order polynomial, and the optimum is reached when the roots occur as two identical complex pairs.

The variables δ and γ, which determine the angular relationship of the damper rod and a principal axis of the main satellite body to the orbital plane, dictate the distribution of mass within the main satellite body because the ratio I_d/I_3 has been fixed for this study. In nearly all cases, the optimum mass distribution was found to be planar, that is, $J_3 = 1$ (see Fig. 2). This mass distribution obviously cannot be achieved in practice. However, when extensible structures, such as the rods shown in Fig. 1, are used to increase the moment of inertia, it is found that the mass distribution is not very different from planar. A practical lower limit for J_3 is expected to be about 0.9. When this limit on J_3 is imposed, it is found that the magnitude of σ_{max} is reduced by roughly 8%.

Once a system has been found which is optimum in some sense, the question of sensitivity to off-design conditions al-

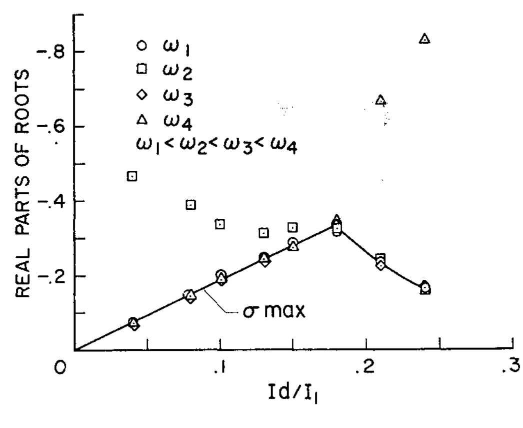


Fig. 3 Real parts of roots of characteristic equation for single-degree-of-freedom damper.

ways arises. Such off-design conditions can result, for instance, from temperature variations, manufacturing tolerances, or deterioration with time. The sensitivity of the damping of the least damped mode to changes in spring constant, damper constant, and the angle (δ-γ) is shown in Fig. 4. This sensitivity is illustrated for three damper-rod inertia ratios. The largest is $I_d/I_1 = 0.18$ which is near the point of maximum damping. The others are for smaller damper-rod inertia ratios, the smallest being $I_d/I_1 = 0.08$. In each case, it is apparent that the point taken as the optimum does represent a stationary value in that variations in any of the variables from this point always cause $\sigma_{\max}$ to increase. It is also apparent that the spring constant C_2 must be within a small percent of the calculated value if the performance is to remain nearly optimum. As the spring becomes weaker ($\Delta C_2/C_2$ is negative) than a certain critical value, an instability is indicated. This means that the destabilizing gravity-gradient torque on the damper rod is sufficient to overcome the spring torque. When the real nonlinear nature of the gravity-gradient torque is considered, the damper rod will merely be found to have a null position other than $\theta_d = 0$. As a consequence (in the absence of disturbing torques), the steady-state values of θ_{v0} and φ_{v0} in this case are also different from zero. The damping is considerably less sensitive to variations in the damping constant and the damper angle than it is to changes in the spring constant. This is particularly true for the smaller values of damper-rod inertia ratio.

It is apparent from the results shown in Fig. 4 that the sensitivity to changes in system parameters can influence the choice of damper-rod inertia ratio. For instance, if C_2 is not known to better than $\pm 5\%$ of the required value, the performance when $I_d/I_1 = 0.18$ is likely to be inferior to that when $I_d/I_1 = 0.13$. If the uncertainty in C_2 is $\pm 10\%$, the performance for $I_d/I_1 = 0.18$ is likely to be no better than for $I_d/I_1 = 0.08$ and can be considerably worse in some instances. Therefore, unless the system parameters are known with great certainty, there is no advantage in selecting the configuration that yields the minimum value of $\sigma_{\max}$. There are also other reasons for not selecting this configuration, as will be illustrated later.

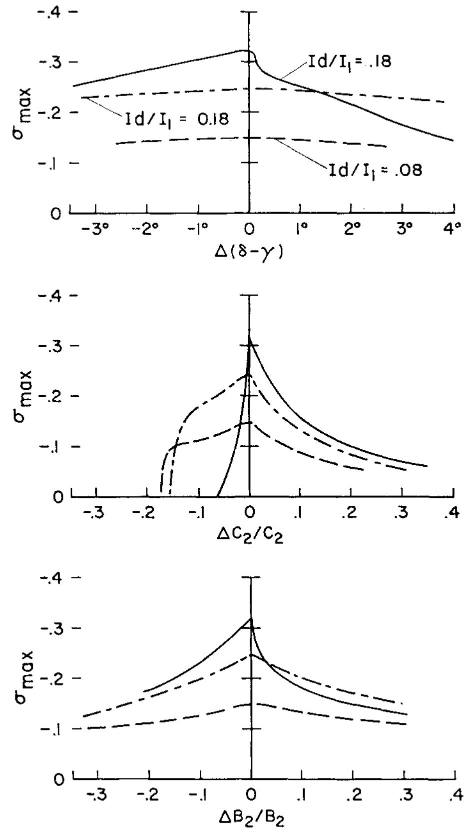


Fig. 4 Sensitivity of damping of least-damped mode to off-design conditions.

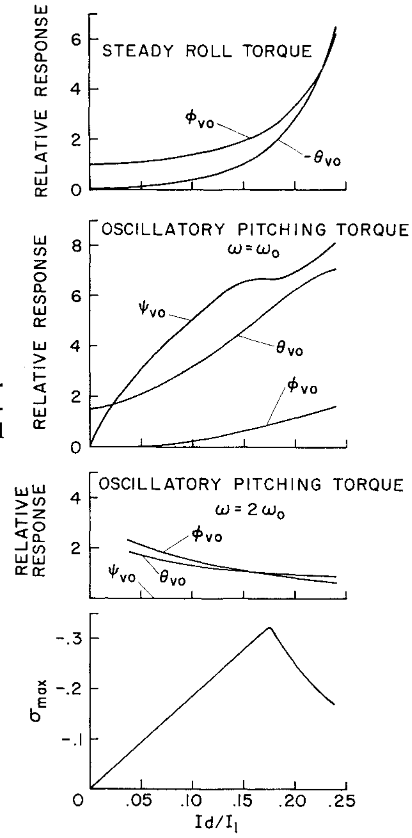


Fig. 5 Relative response to disturbance torques induced by solar pressure.

Adequate damping of the transient motion insures rapid decay of the motion to the steady state determined by the static and periodic disturbance torques. The magnitude of the pointing error caused by the disturbance torques depends to a great extent upon orbital altitude. At the lower altitudes, the gravity gradient can be made to produce restoring torques large enough to limit the errors to a reasonable magnitude. At the higher altitudes, particularly synchronous altitude, large restoring torques cannot be achieved easily. This is because the extensible structures necessary for achieving the required difference in the principal moment of inertia become either prohibitively long or prohibitively heavy.

At the higher orbital altitudes, solar pressure forces are expected to be the principal source of external disturbance. Solar torques occur if the center of solar pressure is not coincident with the center of mass. The solar pressure disturbance produces oscillatory torques about the pitch axis at orbital frequency and its harmonics and steady torques about the roll axis. The variation of the steady-state response to these disturbances with I_d/I_1 is shown in Fig. 5. The system parameters for each value of I_d/I_1 are those found to provide optimum damping of the transient motion. The damping of the transient motion is included in the lower part of the figure for reference.

The term "relative response," used for the ordinate in Fig. 5, requires some explanation. The term means that the response is relative to the response of a slender rod. The slender rod used as a standard of comparison has the same magnitude of I_1 as the satellite and is disturbed by a steady torque of the same magnitude as the amplitude of the disturbance torque in question. Note that $I_d/I_1 = 0$ implies that the mass distribution can be represented by a slender rod. Thus, because I_3/I_d is fixed for this study, I_3 must approach zero as I_d approaches zero. Therefore, the relative responses of a slender rod are found on the ordinate axis and

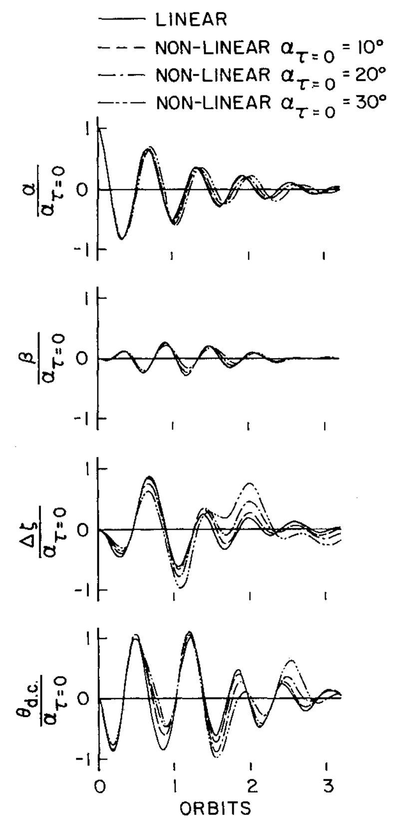


Fig. 6 Comparison of motions calculated by linearized and by complete nonlinear equations of motion.

the roll response to a steady roll torque is unity at this point. Similarly, the relative response to an oscillatory pitching moment is 1.5 when $I_d/I_1 = 0$. This corresponds to the ratio of the response of a rod to a torque of a given amplitude at $\omega = \omega_o$ to that when $\omega = 0$.

A steady roll torque produces errors about both the pitch and roll axes but produces no error in yaw. At the point of maximum damping of the transient motion, $I_d/I_1 = 0.18$, the relative response is roughly 2.6 about the roll axis and 1.8 about the pitch axis. This reduction in static restoring torques occurs because the mass distribution becomes less like that of a rod and more like that of a spheroid as the damper-rod inertia ratio is increased. The gravity torque does, of course, become destabilizing when the moment of inertia about the yaw axis exceeds that about the roll axis.

The reduction in restoring torque which accompanies increases in damper-rod moment of inertia is, in part, the cause of the increase in the relative response to a pitching disturbance torque of orbital frequency. This response increases from 1.5 to almost 6 at the point where the damping of the transient motion is a maximum ($I_d/I_1 = 0.18$). Note also

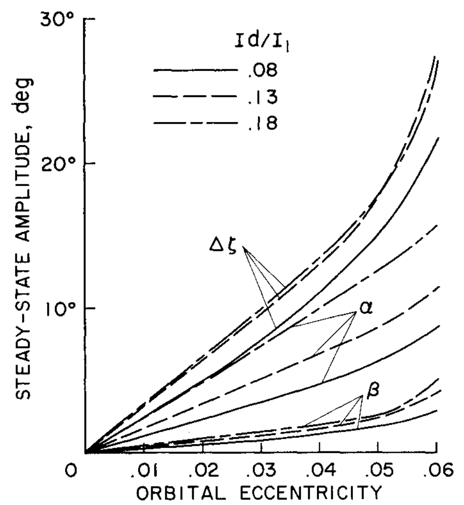
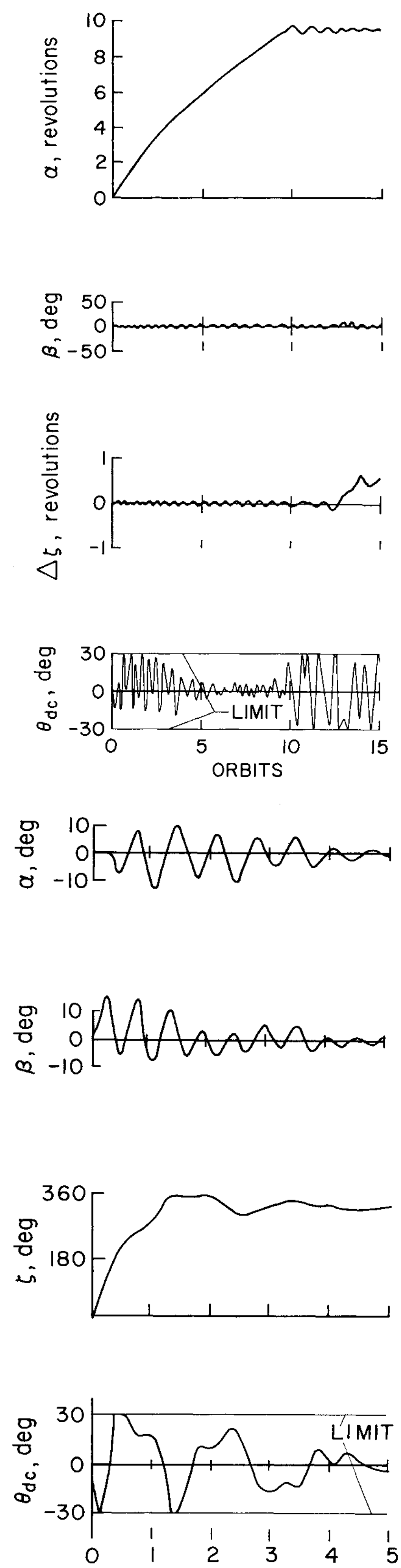


Fig. 7 Effect of orbital eccentricity on steady-state amplitude of satellite motion.



a) Initial pitch velocity in same sense as orbital rate ($\dot{\alpha} = 2\omega_0$)

b) Initial yaw rate $= 2\omega_0$.

Fig. 8 Recovery from tumbling motions; circular orbit, $I_d/I_1 = 0.08$.

that the response in yaw is greater than that in pitch when $I_d/I_1 > 0.02$. The large response in yaw is primarily a resonance effect, since the natural frequency of the satellite in yaw lies near the orbital frequency.

It has been pointed out in unpublished work by the General Electric Company that one of the serious problems with configurations, such as shown in Fig. 1, is thermal bending of the rods. Thermal bending causes unequal pressure forces on opposite tips of the rod, except when the rod is either aligned with, or normal to, the sun line. The presence of this torque has been shown to place an upper limit on the length of the gravity-gradient rods in that a longer rod will increase, rather than decrease, the pointing error. The pitch component of the disturbance caused by this thermal effect occurs at twice orbital frequency. In Fig. 5, the relative responses in roll and pitch to disturbances of this frequency are shown to be about 2 at small values of I_d/I_1 and to diminish to about 1 when $I_d/I_1 = 0.18$.

Large-Angle Motion

Since the procedure for finding optimum configurations is based on the linearized equations of motion, it is important to determine the angular range wherein the linear equations can be used with some degree of confidence. The results of some calculations directed toward this end are summarized in Fig. 6. The satellite is assumed to have an initial error in pitch. In each case where the nonlinear equations of motion are used, the resultant error angles are divided by the initial pitch angle to afford a direct comparison with the results from the linearized equations of motion. The results of these calculations indicate that the linearized equations are adequate for predicting the damping and the coupling of the motion for initial pitch errors of the order of 10°. For larger angles, the coupling of the motion into the yaw axis is predicted with progressively poorer accuracy. However, the envelope of the motions about the other two axes, that is, the damping of the pitch and roll motion, is predicted quite well for initial pitch angles as large as 30°.

Orbital eccentricity excites steady-state oscillations about all three axes. The maximum amplitude of these oscillations, measured from the steady-state, torque-free attitude for a circular orbit, is shown in Fig. 7. As might be anticipated, there are similarities between the response to orbital eccentricity and the response to a 1-cycle/orbit torque about the pitch axis. In each case, the largest response occurs about the yaw axis and the steady-state amplitudes increase as the damper-rod inertia ratio I_d/I_1 increases. The response in pitch is particularly sensitive to damper-rod inertia ratio, nearly doubling as I_d/I_1 is increased from 0.08 to 0.18.

The primary conclusion that can be drawn concerning the effects of orbital eccentricity is that it is imperative to launch a passive gravity-gradient satellite into a very nearly circular orbit if sizable steady-state pointing errors are to be avoided. For example, when $I_d/I_1 = 0.13$, an eccentricity of only 0.03 will cause steady-state oscillations of about 10°, 5°, and 1° in yaw, pitch, and roll, respectively. A decrease in the damper-rod inertia ratio reduces the error caused by a given eccentricity but only at the expense of decreasing the damping of transient motions.

A passively oriented satellite might be required to stabilize from some random initial position or possibly an initial angular rate large enough to cause a tumbling motion. The size of these initial errors depends upon the launch technique employed. For purposes of demonstrating recovery from possible adverse launch conditions, computations have been made to illustrate recovery from initial tumbling motions. The results are shown in Fig. 8. In each case, the damper-rod inertia ratio I_d/I_1 is 0.08, there are no external disturbance torques, and the orbit is circular. The damper-rod motion has been limited by hard stops at $\theta_{dc} = \pm 30°$.

The results shown in Fig. 8a illustrate recovery from an initial tumbling motion about the pitch axis. The rate, relative to inertial space, is three times the orbital rate and in the same direction. For this initial condition, the satellite performs $9\frac{1}{2}$ revolutions about the pitch axis before oscillatory, rather than tumbling, motion begins. At first there is considerable motion of the damper rod which tends to subside to what might be considered a steady-state oscillation of the damper rod during a tumbling motion. Finally, when sufficient energy has been removed from the system to cause oscillatory motion, the damper rod performs wide excursions and the system damps rapidly. Note that the yaw angle changes by 180° during the final stages of stabilization. This, and the fact that the satellite rotates $9\frac{1}{2}$ times during the initial stabilization, illustrates a fundamental problem of passive gravity-gradient satellites, that is, they are bistable about each axis. There are, therefore, four distinct final orientations possible.

The previous example illustrates recovery from tumbling about the pitch axis. It might be suspected that tumbling about the yaw axis can present greater difficulties, since the restoring torque is least about this axis. Such is not the case, as illustrated by the time history shown in Fig. 8b. The satellite performs only one revolution following an initial angular rate of $2\omega_o$ about the yaw axis. It can be reasoned that the angular rate subsides more rapidly in this case because the yaw axis is the axis of least inertia. A given angular rate therefore represents less energy to be dissipated by the damper than the same initial rate applied about any transverse axis.

Concluding Remarks

The analysis described leads to an understanding of the basic requirements of an effective passive gravity-gradient orientation scheme of minimum weight and complexity. The results show that inertial coupling permits all transient motions to be damped with a single auxiliary body that has a single degree of freedom.

The auxiliary body was assumed to be a slender rod constrained to move in a vertical plane and to be in the horizontal plane when in the null position. Because of these constraints, the full potential of inertial coupling may not have been realized. Performance might be improved if the only restriction imposed were that the mass distribution be physically possible. Then, the plane of motion and the null position of the damper rod would be arbitrary and subject to optimization. In this instance, all three cross products of inertia would come into play and produce inertial coupling to enhance the performance of the single-degree-of-freedom damper.

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